APRIL/MAY 2024

23PMA11 — ALGEBRAIC STRUCTURES

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Equivalence relation.
2. Define Normalizer.
3. Define Normal subgroup.
4. Define Modulas.
5. Define Nilpotent.
6. Define Invariant.
7. Define Jordan Block.
8. Define characteristic polynomial.
9. Define Trace.
10. Define Unitary Transformation.

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SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Sylow's Theorem. (First)

17. Prove that, Let $G(x)$, be an irreducible Polynomial of degree P,P prime over the field Q of rational numbers. Suppose that $q(x)$ has exactly two non real roots in the field of complex numbers. Then the Galoi's group of $q(x)$ over Q is S_P, the symmetric group of order P. Thus the splitting field of $q(x)$ over Q has degree $P!$ over Q.

18. Show that two nilpotent linear transformation are similar if and only if they have the same invariants.

19. Prove that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisions.

20. Prove that the linear transformation T or V is unitary if and only if makes an orthonormal basis of V into an orthonormal basis of V.

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SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions.

11. (a) If P is a prime number and $P | o(G)$ then show that G has an element of order P.

Or

(b) If $P \wedge m \mid o(G)$, $P^{m+1} \nmid o(G)$ then prove that G has a subgroup of order P^m.

12. (a) Show that every finite abelian group G is the direct product of cyclic groups.

Or

(b) Let $G = S_n$, when $n \geq 5$, then show that $G^{(k)}$ for $k = 1, 2, \ldots$ contains every s-cycle of S_n.

13. (a) Prove that if $w \subset V$ is invariant under T, then T induces a linear transformation T on $\frac{V}{W}$, Defined by $(v+w)T = vT+w$, If T satisfies the polynomial $q(x) \in F[x]$, then so does T. If $P_1(x)$ is the minimal polynomial for T over F and if $P(x)$ is that for T, then $\frac{P_1(x)}{P(x)}$.

Or

(b) Prove that if $T \in A(V)$ is nilpotent, then $\alpha_0 + \alpha_1 T + \ldots + \alpha_m T^m$, when the $\alpha_i \in F$, is invertible if $\alpha_0 \neq 0$.

14. (a) Prove that Let $t \in A_F(V)$ have all its distinct characteristic roots $\lambda_1, \lambda_2, \ldots \lambda_k$ in F. Then a basis of V can be found in which the matrix T is of the form $\begin{pmatrix} F_1 & & \\ & F_2 & \\ & & F_3 \end{pmatrix}$ where each $F_1 = \begin{pmatrix} B_{i1} & & \\ & \ddots & \\ & & B_i r_i \end{pmatrix}$ and where $B_{i1}, \ldots B_i r_i$ are basic Jordan blocks belonging λ_i.

Or

(b) Show that, suppose two matrices A, B in F_n are similar in K_n where K is an extension of F. Then A and B are already similar in F_n.

15. (a) Prove that, If F is of characteristic o and S and T in $A_F(V)$, are such that ST and TS commutes with S, then $ST - TS$ is nilpotent.

Or

(b) If A is invertible, then show that for all B, $\det(AB\,A^{-1}) = \det B$.